

Preface

This book gathers most of the results of my work on proof-theoretical tools of the logic of questions known as Inferential Erotetic Logic, especially those focused on the method of Socratic proofs.

Chapter 1 is an overview of the main achievements that belong both to the area of the logic of questions and proof theory. I describe the most important frameworks and indicate the place of my research against this background.

Chapter 2 presents the main characters: erotetic calculi for classical logic, both propositional and first-order, in various variants, and erotetic calculi for the class of the so-called basic modal logics. The method of Socratic proofs was invented by Andrzej Wiśniewski; it is a method of transforming questions concerning logical properties and relations such as validity or derivability, which is at the same time a proof method for an underlying logic. The method certainly lies at the very centre of the intersection of the logic of questions and proof theory. The erotetic calculi presented here are notational variants of the original calculi. I have decided to work on variants of the method with the unified notation consequently extended to the quantifier level, but I present both versions of the calculi. The chapter also contains an analysis of admissibility and derivability of erotetic rules which is basically of my authorship, as well as the algorithm for proof-search in the erotetic calculi for the first-order logic (though the idea is derived from Fitting, 1983).

The part of Chapter 2 devoted to modal logics is based mainly on my monograph (Leszczyńska, 2007), but the erotetic calculi are presented in a new version; the difference concerns the definitions of provisos of applicability of modal rules. I compare the two approaches in detail in Appendix A.

Chapter 3 presents the natural, indispensable environment of the method of Socratic proofs. Erotetic calculi are calculi of questions, and should be analysed as such; therefore, in this chapter, I recall the main ideas of Minimal Erotetic Semantics developed for this purpose by Andrzej Wiśniewski. The version of the notion of admissible partition for the first-order case which is contained in this book is new, while the def-

initiation of admissible partitions for modal logics is also of my authorship, but has been already presented in Leszczyńska, 2007. The discussion concerning erotetic implication is based on my research report Leszczyńska-Jasion, 2017. The remaining parts of Chapter 3, concerning the proof-theoretical settlement of the multiple-conclusion entailment relation, are published here for the first time.

Chapter 4 may be viewed as the main part of this monograph. It is focused on exploring relations between various proof systems by defining the so-called “translations” between them. More specifically, the chapter shows how to translate a Socratic proof—that is, a sequence of questions of a certain kind—into a proof of the relevant formula in other proof systems, such as sequent calculus or analytic tableaux. Sections 4.1 and 4.2 are based on Leszczyńska-Jasion et al., 2013. The case of analytic tableaux described in Sections 4.5 and 4.6 was prepared by myself earlier in the form of a research report (Leszczyńska-Jasion, 2015). During work on this report I used the original versions of erotetic calculi by Wiśniewski, and I was planning to modify the results contained in Leszczyńska-Jasion, 2015 *via* the notational differences introduced here. However, I then realised that some specific details of the translation would be lost, namely the games one has to go into with the apparently transparent double-classical-negation, and therefore decided to leave it in the original version. The last subsection 4.7 is partially based on Chlebowski and Leszczyńska-Jasion, 2015. The remaining parts of Chapter 4, namely Sections 4.3 and 4.4, are published for the first time.

The various parts of this book were written over the last, more or less, five years. Another monograph was also written during this period—a doctoral dissertation by Szymon Chlebowski (Chlebowski, 2018); I was the thesis’ technical supervisor, hence many parts of the two monographs have been discussed jointly. Therefore there is a certain convergence of topics and techniques used in the two monographs. Although the results are properly separated, this may give the impression that one of the books complements the other. This is indeed the case.

I have strived to write the monograph in a self-contained way. Nonetheless, some basic background in algorithms (like the ability to read pseudocode) may be needed, and basic knowledge in the field of modal logics is necessary to read Chapter 4. If needed, the reader should consult Corman et al., 2009; Hughes and Cresswell, 1996.

Needless to say, I feel solely responsible for all the mistakes this book contains.