Contents

Preface **9**

Part I Logical aspects **13**

Chapter 1 Mathematical theories and their models **15**
1.1 Theories in polymathematics and monomathematics . . . 16
1.2 Types of models and their comparison 20
1.3 Classification and representation theorems 32
1.4 Which mathematical objects are standard? 35

Chapter 2 Historical remarks concerning extremal axioms **43**
2.1 Origin of the notion of isomorphism 43
2.2 The notions of completeness 46
2.3 Extremal axioms: first formulations 49
2.4 The work of Carnap and Bachmann 63
2.5 Further developments 71

Chapter 3 The expressive power of logic and limitative theorems **73**
3.1 Expressive versus deductive power of logic 73
3.2 Metalogic and metamathematics 76
3.3 Limitative theorems 84
3.4 Abstract logics and Lindström's theorems 88
3.5 Examples 90

Chapter 4 Categoricity and completeness results in model theory **103**
4.1 Goals of model theory 103
4.2 Examples of categoricity and completeness results 107
4.3 Ultraproducts 108

4.4 Definability 111
4.5 A few words about types 113
4.6 Special models 116
4.7 Classifying theories 118

Part II Mathematical aspects **123**

Chapter 5 The axiom of completeness in geometry, algebra and analysis **125**
5.1 Geometry 127
5.2 Algebra and analysis 137
5.3 Axiom of continuity and its equivalents 142
5.4 Generalizations and isomorphism theorems 144
5.5 Degrees of infinity, pantachies and gaps 152
5.6 Infinitesimals and non-Archimedean structures 154
5.7 Continua in topology 161

Chapter 6 The axiom of induction in arithmetic **163**
6.1 A few historical remarks 163
6.2 Definitions of finiteness 171
6.3 First-order arithmetic 173
6.4 Second-order arithmetic 177
6.5 Non-standard models of arithmetic 178
6.6 Transfinite induction in set theory 183

Chapter 7 Two types of extremal axioms in set theory **187**
7.1 Introductory remarks 187
7.2 Zermelo: two axiomatizations of set theory 192
7.3 Axioms of restriction 198
7.4 Large cardinal axioms 213
7.5 Sentences independent from the axioms 219

Part III Cognitive aspects **221**

Chapter 8 Mathematical intuition **223**
8.1 Philosophical remarks 223
8.2 Research practice 232
8.3 Teaching practice 250

A final word **267**

Bibliography **271**

Author index **293**

Subject index **301**